On nonexistence of Kenmotsu structure on Kirichenko–Uskorev-hypersurfaces of Kählerian manifolds

Galina A. Banaru
(Chair of Applied Mathematics, Smolensk State University, Przhevalski str., 4, Smolensk – 214 000, Russian Federation)
E-mail: mihail.banaru@yahoo.com

1. The almost contact metric (*acm*-) structure is one of the most important differential-geometrical structures on manifolds. As it is known [1], an almost contact metric structure on a odd-dimensional manifold N is a system $\{\Phi,\, \xi,\, \eta,\, g\}$ of tensor fields on this manifold, where Φ is a tensor of type $(1, 1)$, ξ is a vector, η is a covector and $g = \langle\cdot,\, \cdot\rangle$ is a Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1;\ \Phi(\xi) = 0;\ \eta \circ \Phi = 0;\ \Phi^2 = -id + \xi \otimes \eta;$$
$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta(X)\eta(Y),\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of smooth vector fields on N. As one of the most meaningful and interesting *acm*-structure we mark ut the Kenmotsu structure that is defined by the following condition [2]:

$$\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\, \xi - \eta(Y)\Phi X, \quad X, Y \in \aleph(N),$$

In [3], V. F. Kirichenko and I. V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type. V. F. Kirichenko and I. V. Uskorev have also proved that their structure is invariant under canonical conformal transformations [3].

Evidently, a trivial example of Kirichenko–Uskorev structure is the cosymplectic structure, and as a non-trivial example we can consider the Kenmotsu structure.

2. Now let us consider the *acm*-structure induced on a hypersurface N of a Kählerian manifold M^{2n}, $n \geq 3$. The Cartan structural equations of such *acm*-structure are the following [4]:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^\alpha_\beta \omega^\beta \wedge \omega + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega,$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma^\beta_\alpha \omega_\beta \wedge \omega - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega,$$
$$d\omega = -i\sigma^\alpha_\beta\, \omega^\beta \wedge \omega_\alpha + i\sigma_{n\beta}\omega \wedge \omega^\beta - i\sigma^\beta_n \omega \wedge \omega_\beta.$$

Here σ is the second fundamental form of the immersion of N into M^{2n}; $\omega_\alpha = \omega^{\hat{a}}$; $\alpha,\, \beta = 1\,, \ldots,\, n-1$; $\hat{a} = a + n$.

Taking into account the results on the matrix of the second fundamental form [5], we obtain the first Theorem.

Theorem 1. *The Cartan structural equations of Kirichenko–Uskorev acm-structure induced on a hypersurface of a Kählerian manifold M^{2n}, $n \geq 3$ are the following:*

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega;$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega;$$
$$d\omega = 0.$$

Comparing these equations with well-known Cartan structural equation of a Kenmotsu structure [1]

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + \omega \wedge \omega^\alpha;$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + \omega \wedge \omega_\alpha;$$
$$d\omega = 0,$$

we obtain our second result.

Theorem 2. *Krichenko–Uskorev almost contact metric structure induced on a hypersurface of a Kählerian manifold M^{2n}, $n \geq 3$, cannot be a Kenmotsu structure.*

Note that the presented Theorems develop some results on hypersurfaces of Kählerian manifolds [5], [6].

References

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